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Bidirectional Evolutionary Method for Stiffness Optimization

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Evolutionary structural optimization (ESO) method was originally developed based on the idea that by systematically removing the inefficient material, the residual shape of the structure evolves toward an optimum. This paper presents an extension of the method called bidirectional ESO (BESO) for topology optimization subject to stiffness and displacement constraints. BESO allows for the material to be added as well as to be removed to modify the structural topology. Basic concepts of BESO including the sensitivity number and displacement extrapolation are proposed and optimization procedures are presented. Integrated with the finite element analysis technique, BESO is applied to several two-dimensional plane stress problems. Its effectiveness and efficiency are examined in comparison with the results obtained by ESO. It is found that BESO is more reliable and computationally more efficient than ESO in most cases. Its capability and limitation are discussed.

Nomenclature

C = mean compliance
E = Young's modulus
$\boldsymbol{K}^i$ = element stiffness matrix
t = thickness of plate
u_j = displacement at the constrained location
u^* = limit of the displacement constraint
$\boldsymbol{u}^i$ = element displacement vector due to real load
$\boldsymbol{u}^{ij}$ = element displacement vector due to unit virtual load acting at the location of the displacement constraint
W = weight of current structure
$W_{\max}$ = weight of maximum structure
W_{obj} = objective weight
$W_{\text{obj}}^{\min(1)}$ = first local minimum of the objective weight
$W_{\text{obj}}^{\min(2)}$ = second local minimum of the objective weight
W_{opt} = weight of optimal topology
W_0 = weight of structure of the full design area
W^* = target weight
α = sensitivity number
Δ = increment
ν = Poisson's ratio

Introduction

TOPOLOGY optimization, in which the structural connectivity is sought in addition to the member size and structural shape, has enjoyed rapid progress recently. Many efforts have been devoted to finding the optimal topologies of two-dimensional continuous structures. Various optimisation methods integrated with the finite element analysis have been proposed. Bendsøe and Kikuchi[1] presented a homogenization method in which the topology optimization was transformed to an optimal material distribution problem by treating the structural elements as porous media at the microscopic lever. A simpler technique named the soft kill option method was recently formulated by Mattheck[2] to solve the fully stressed design problem. The soft kill option keeps changing the elasticity modulus of each element according to its stress level and those with the lowest level are killed.

Evolutionary structural optimization (ESO) is an alternative optimization method also using the finite element anslysis.[3] The basic concept is that by systematically removing the inefficient material, the residual shape of the structure evolves toward an optimum.[3] For problems of stress-based constraint, the material inefficiency can be easily evaluated by the low element stress level. Gradually removing those elements can result in a design of more uniform stress. In contrast, there is a class of problems using the sensitivity analysis to measure the element efficiency. Optimization with the stiffness/displacement, buckling load, and frequency constraints falls into this category. The sensitivity analysis is the same as that in the mathematical programming and optimality criterion algorithms except that the design variable is discrete in ESO. The sensitivity analysis or the element stress requires the information on the results of the finite element analysis. Therefore, the implementation of ESO method consists of the finite element analysis, element efficiency evaluation, and structural modification. A common procedure exists for problems of different constraints as these three parts are physically independent and only the second one varies with the nature of the constraint.[3–7]

A more recent advancement in ESO was the formulation of the methodology of bidirectional ESO (BESO).[7] As ESO only considers removing elements and those removed ones cannot be brought back in the later evolution, an oversized initial design area is required to ensure that the final design is represented by adequate elements. In some structures the optimization is misled because of an inappropriately defined initial area.[7] BESO allows for the elements to be added as well as removed, so that the final optimum can be reached regardless of how good or bad the initial design is defined. Indeed, BESO is more flexible in choosing the initial design, and any design complying with the loading and boundary conditions can be used as an initial design. Quite often, it is convenient to use the simplest design connecting the loads and supports, which can be automatically generated in constructing the finite element model. As the finite element model is significantly reduced in BESO, it has potential to save the computing cost. BESO has been effectively applied to the uniform stress design of two-dimensional and three-dimensional continua,[7,8] and the work was further developed for the frequency optimization.[9]

The BESO method for topology optimization of two-dimensional continuous structures with stiffness and displacement constraints is addressed in this paper. Theoretical aspects such as the sensitivity number and displacement extrapolation are first investigated. The BESO procedure is illustrated through several examples, and the results of BESO and ESO are compared.

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Basic Concepts in Stiffness Optimization

Element Sensitivity

In a structure discretized by a finite element mesh, when the ith element is removed or added, the displacement u_j at a specified node or the mean compliance C changes as follows[3]:

$$\Delta u_j = \pm \boldsymbol{u}^{ij} \boldsymbol{K}^i \boldsymbol{u}^i \tag{1a}$$

$$\Delta C = \pm \boldsymbol{u}^i \boldsymbol{K}^i \boldsymbol{u}^i \tag{1b}$$

The unit virtual load resulting in the displacement $\boldsymbol{u}^{ij}$ is of the same direction as the displacement constraint. Plus and minus signs are for the removed and added elements, respectively.

If the objective is to reduce the absolute value of the displacement $|u_j|$ or the mean compliance C, the sensitivity number is defined as

$$\alpha = \operatorname{sign}(\Delta u_j, u_j) \tag{2a}$$

$$\alpha = \Delta C \tag{2b}$$

The sensitivity number α indicates the contribution of an element removal or addition to the displacement or mean compliance. Removing elements with the smallest sensitivity or adding the elements with the largest will make the structure evolve to a stiffer design.

Displacement Extrapolation

Because Eqs. (1a) and (1b) require the information of displacement vectors $\boldsymbol{u}^{ij}$ and $\boldsymbol{u}^i$, it is necessary to extrapolate the displacements for the added elements. Take a four-node square element in which the bilinear displacement function is adopted. Assume that the shape shown in Fig. 1 is obtained during optimization, which satisfies all of the kinematic boundary conditions. Elements drawn with dashed lines are potential added elements, which are attached to the external and internal boundaries of the structure. They are classified into three types according to the number of nodes of known displacements: A2, with known displacements at two nodes; A3, with known displacements at three nodes; and A4, with known displacements at four nodes.

As shown in Fig. 2, the displacements vary linearly along the element edge, so that the unknown displacements are calculated for element types A2 and A3, respectively, as follows:

$$u_3 = 2u_4 - u_B \tag{3}$$

$$u_3 = u_2 + u_4 - u_1 \tag{4}$$

Topology Performance

The evaluation of the performance of a candidate design and comparison of two intermediate topologies in the evolution process

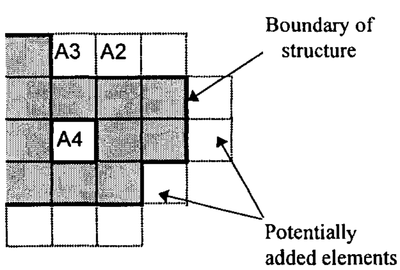
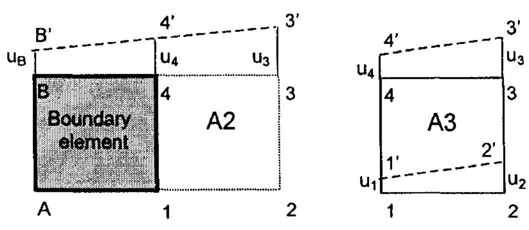


Fig. 1 Types of potentially added elements.



Element type 2 **Element type 3**

Fig. 2 Displacement extrapolation.

are relevant to the scaling of the design. The problem is posed as to convert an intermediate design by scaling the design variables so that the resulting design has the displacement (or mean compliance) limit u^* while maintaining the current topology. In doing so, we assume that the stiffness matrix is the linear function of the zth order of the design variable and the structural weight is proportional to the design variable, e.g., in the case of two-dimensional plane stress problems where the plate thickness is taken as the design variable, $z = 1$, and for plating bending problem, $z = 3$.

The scaling ratio μ is defined as

$$\mu = \left(u_j / u^*\right)^{1/z} \tag{5}$$

All design variables of the structure are scaled by μ. Therefore, the resulting design has a weight:

$$W_{\text{obj}} = \mu W = W\left(u_j / u^*\right)^{1/z} \tag{6}$$

A smaller value of objective weight means a better topology. The objective weight is expected to keep decreasing during optimization, and when it starts to increase, the optimization direction needs to be changed or the optimization may be terminated. This will be discussed further in the following section as well as with examples.

Evolution Procedures

Like most optimization methods, BESO proceeds in an iterative manner. The finite element model is changing during iterations so a remeshing procedure is often required by many optimization methods. The element remeshing, however, is avoided in BESO and ESO by using a strategy of element property number. The existing elements in the structure are assigned a nonzero property number according to the physical and geometrical properties. If an element is removed during the optimization, its property number switches to zero. In finite element analysis, elements tagged zero will be treated as absent ones and be ignored when the stiffness matrix is assembled. Likewise, an added element will be considered in the structural analysis by changing its property number from zero to nonzero. Based on such a strategy, the finite element mesh is only generated once in the beginning and used throughout the evolution.

The procedures of BESO with stiffness and displacement constraints are outlined as follows:

1) Define the full design area that the structure can occupy.

2) Discretize the area with a finite element mesh. Specify the initial design consisting of connecting elements that transfer the loads to the supports. All elements in the full design area not marked as connecting elements are assigned a zero property number.

3) Solve the static problem to obtain the structure response.

4) Locate the potentially added elements of types A2, A3, and A4 and calculate their nodal displacements by Eqs. (3) and (4).

5) Calculate the sensitivity number α for existing elements and potentially added elements by Eqs. (1a–2b).

6) Modify the elements according to the sensitivity number and change the property number accordingly.

7) Repeat steps 3–6 until the conditions for terminating the evolution are satisfied.

As for step 6, instead of removing or adding a large amount of material at one time, only a small number of elements are modified in a single iteration. The number of modified elements is determined by the modification ratio (MR), addition ratio (AR), and the reference structure, which can be the full design area or the current structure. For example, if the full design area with 2000 elements is chosen as the reference structure and MR = 1% and AR = 0.6 are assumed; 2000 × 1% = 20 elements in the current structure are modified, among which 20 × 0.6 = 12 elements are added and 8 elements are removed. As a result, the structure after modification has four elements more than the old one and the weight increases.

For step 7, three schemes are developed according to the design requirements.

The first case is to find the best optimum. The best optimum is simply the topology with the minimum objective weight. First, set AR > 0.5 so that the added elements outnumber the removed ones and the structural weight increases. Although the objective weight W_{obj} keeps a decreasing trend, probably with the exception of one or two iterations, it may change to an increasing trend after some iterations. If it keeps increasing continuously for 10 iterations, it is assumed that its first local minimum $W_{obj}^{min(1)}$ has been reached and the structure has grown to its maximum weight W_{max} and is termed maximum structure. Second, set AR < 0.5. Both the weight W and the objective weight W_{obj} decrease. Likewise, if after some iterations W_{obj} has a continuous increase from a second local minimum $W_{obj}^{min(2)}$, the evolution terminates and the topology corresponding to $W_{obj}^{min(2)}$ with a weight W_{opt} is taken as the best optimal design.

Case 2 is to specify the displacement limit u^*. First, the same scheme as in case 1 is used. Then set AR < 0.5. The weight decreases and the evolution terminates when the specified displacement reaches the limit.

Case 3 is to specify the target weight W^*. The problem of designing the structure of maximum stiffness for a given amount of material is equivalent to that of minimizing the weight subject to the stiffness constraint, which has been dealt with in case 1. First, the same scheme as in case 1 is used except that AR is keeping a value of greater than 0.5, so that the structure weight increases until it reaches W_{max} or W^*, whichever is larger. Second, set AR < 0.5, the structure weight decreases until W^* is reached. Third, set AR = 0.5, the same number of elements are removed and added thus the weight is kept constant. The evolution process terminates when the difference in the displacement or mean compliance between two consecutive iterations becomes negligible.

Examples

Four examples are used to test the BESO procedure for stiffness and displacement optimization on a Pentium 200 PC with 32 MB of RAM. The two-dimensional structures in these examples are under the plane stress condition and a four-node square linear element is used in the finite element analysis. The Poisson's ratio ν is assumed to be 0.3 in all examples. As for the parameters of BESO, MR = 1% and AR = 0.75 or 0.25 are assumed and the full design area is taken as the reference structure. All examples are treated equally by both ESO and BESO and the results are compared on the basis of the same finite element mesh and the same modification ratio.

Example 1: Short Cantilever Beam

Figure 3 gives the dimensions and loads of a cantilever beam. The thickness $t = 0.001$ m and $E = 207$ GPa are assumed. The load P is applied at the middle of the free end, where the displacement constraint is imposed downward with a limit $u^* = 0.5$ mm. The full design area is divided into 48 × 30 square elements. The optimization begins from the initial design consisting of two rows of connecting elements between the support and the load, as shown in Fig. 4. The black dots represent the elements that do not physically exist in the initial design. By following the preceding procedures, the

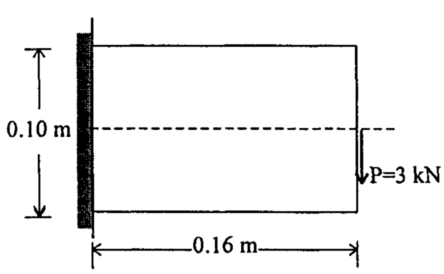


Fig. 3 Cantilever beam.

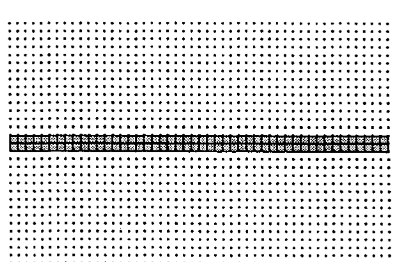
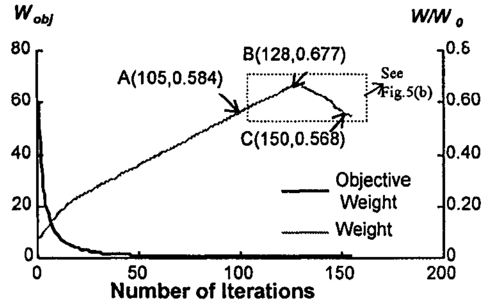


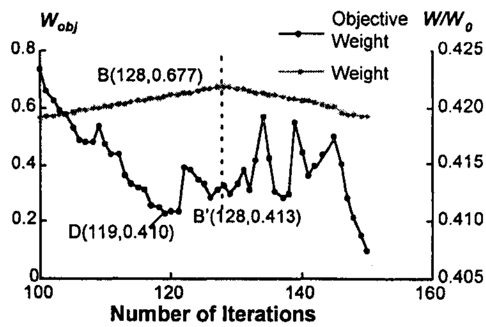
Fig. 4 Initial design (cantilever beam).

Table 1 Comparison of ESO and BESO (cantilever beam)

Type of optimization	Iterations	W/W_0, %	u, mm	W_{obj}	Time, min
ESO	44	57.22	0.501	0.4120	21
BESO	105	58.47	0.498	0.4183	—
	150	56.81	0.496	0.4073	43



a) Iterations 1–155



b) Iterations 100–155

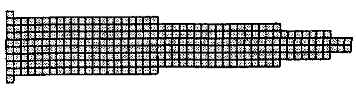
Fig. 5 Evolutionary history of the structural weight and objective weight.

evolutionary histories of the structural weight and objective weight are obtained, as shown in Fig. 5. Some topologies in evolution are given in Fig. 6.

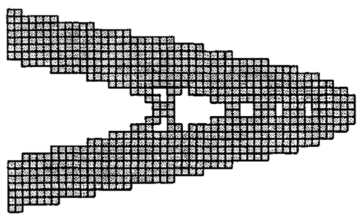
As shown in Fig. 5a, the objective weight reduces dramatically while the structural weight increases in the first 20 iterations. The history of the structural weight is divided into ascending and descending stages at point B. Point B is determined by the objective weight, which is detailed in Fig. 5b. After a relatively stable decrease for about 100 iterations, the objective weight begins to increase from a value of 0.410 at the 119th iteration (point D) up to 0.413 for about 10 iterations. Such a continuous increase indicates that the growth of structural weight is no longer effective and the structure has been built up to its maximum weight W_{max}. The topology corresponding to W_{max} is given in Fig. 6d.

Points A and C in Fig. 5a represent the weight of two structures with the prescribed displacement limit reached at the ascending and descending stages, respectively. Their topologies are shown in Figs. 6c and 6e. The latter has less material than the former and thus it is the optimal solution to this example. In fact, the structure in the ascending stage has not been fully developed and its solution is not reliable even if it complies with the displacement requirement. The point that the optimum can more reliably be found in the descending stage is also applicable to the other two cases where the optimal design with the prescribed weight or the best optimum is the design target.

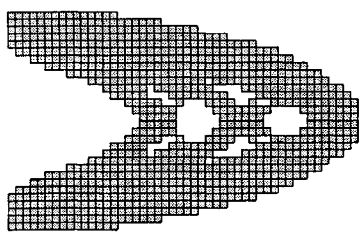
Table 1 compares the results obtained by ESO and BESO. BESO yields a lighter design for the same displacement constraint. As for the computational time, in this example, BESO involves considerably more iterations than ESO and is less efficient. This is because the maximum structure has a large portion of material of the full design area (about 68%).



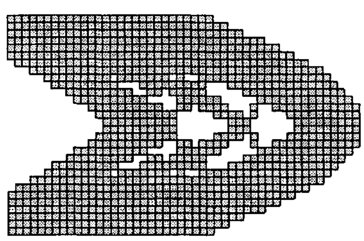
a) Iteration 25: $W/W_0 = 21.25\%$, $W_{obj} = 5.170$



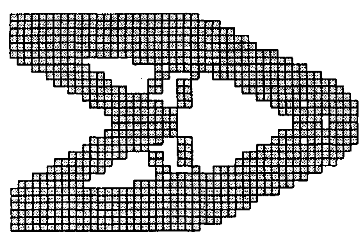
b) Iteration 70: $W/W_0 = 44.03\%$, $W_{obj} = 0.5853$



c) Iteration 105: $W/W_0 = 58.47\%$, $W_{obj} = 0.4183$, $u = 0.498$ mm



d) Iteration 128: $W_{max}/W_0 = 67.78\%$, $W_{obj} = 0.4132$



e) Iteration 150: $W/W_0 = 56.81\%$, $W_{obj} = 0.4073$, $u = 0.496$ mm

Fig. 6 Topologies in evolution (cantilever beam).

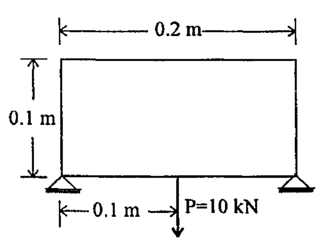


Fig. 7 Michell-type structure.

Example 2: Michell-Type Structure

The dimensions of the design domain, the loading, and supporting conditions are given in Fig. 7. The thickness of the plate $t = 0.001$ m and $E = 210$ GPa are assumed. The full design area is divided into 80×40 totaling 3200 elements. The design objective is to minimize the displacement u at the mid-span for a prescribed target weight W^*. Two weight cases where $W_1^* = 30\%\ W_0$ and $W_2^* = 50\%\ W_0$ are considered. Figure 8 gives the initial design that includes one row of elements at the bottom.

Optimum topologies corresponding to the two weight limits are given in Figs. 9 and 10. It is seen that topologies obtained by BESO and ESO are similar in the outer arch but different in the inner configuration. BESO yields designs of smaller height than those of ESO. The displacement and objective weight by two methods are also different, as shown in Table 2. Differences displayed in the preceding results can be explained by a feature of the ESO method as a ground structure approach, i.e., the initial design does affect the final optimal solution. This point has also been observed in other methods using the ground structure approach.[10] In studying the effect of the initial design on the performance of ESO and BESO,[6,11] it is found that in most cases although the difference in topologies can be relatively distinct, the objective weight is very close with a slight difference ranging within 3%.

As for the solution time, BESO is more efficient for case 1 than for case 2. In case 1, a small target weight is specified and the

Table 2 Comparison of ESO and BESO (Michell-type structure)

Types of optimization	Iterations	W/W_0, %	u, mm	W_{obj}	Time, min
		$W_1^* = 30\%\ W_0$			
ESO	70	30.11	0.737	0.710	100
BESO	95	30.00	0.677	0.649	65
		$W_2^* = 50\%\ W_0$			
ESO	50	49.15	0.492	0.770	80
BESO	109	50.00	0.467	0.749	85

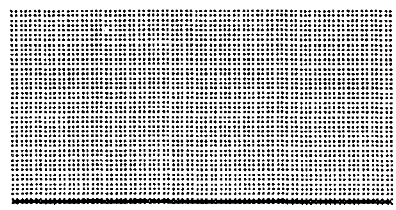
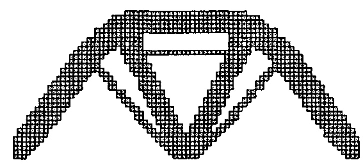
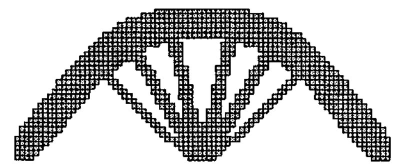


Fig. 8 Initial design (Michell-type structure).

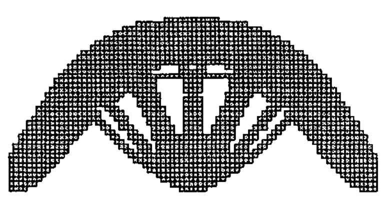


ESO: $W_{obj} = 0.710$

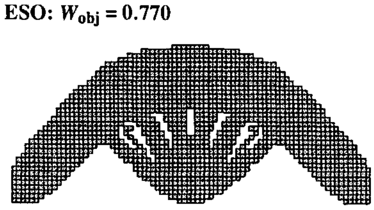


BESO: $W_{obj} = 0.649$

Fig. 9 Optimum: $W_1^*/W_0 = 30\%$.



ESO: $W_{obj} = 0.770$



BESO: $W_{obj} = 0.749$

Fig. 10 Optimum: $W_2^*/W_0 = 50\%$.

Table 3 Comparison of ESO and BESO (lever arm)

Type of optimization	Iterations	W/W_0, %	C, N · m	W_{obj}	Time, min
		Case 1			
ESO	79	18.2	5.95	0.297	69
BESO	59	17.9	5.86	0.288	18
		Case 2			
ESO	80	16.5	1.766	1.400	289
BESO	115	14.5	1.874	1.413	80

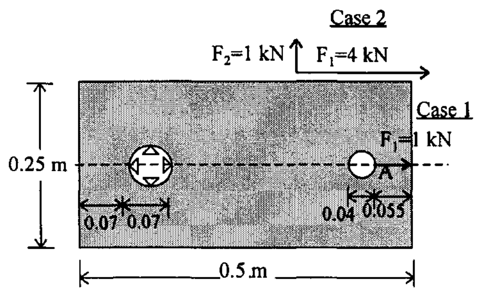


Fig. 11 Design area (lever arm).

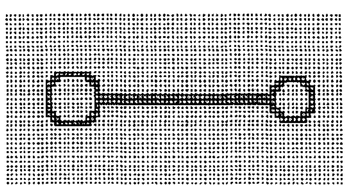
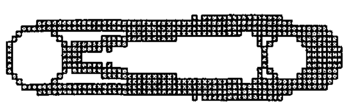
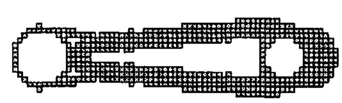


Fig. 12 Initial design (lever arm).



ESO: $W_{opt}/W_0 = 18.2\%$, $W_{obj} = 0.297$



BESO: $W_{opt}/W_0 = 17.9\%$, $W_{obj} = 0.288$

Fig. 13 Optimum topologies (lever arm under tension loading).

maximum structure is also comparably smaller than the full design area (about 37%). Therefore, the computational time is saved because of the smaller finite element model in BESO. In case 2, though the model size in BESO is not as large as that in ESO, BESO involves significantly more iterations and therefore it requires more computing efforts.

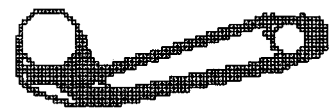
Example 3: Lever Arm

The mechanical behavior of a lever arm is represented by the model in Fig. 11. The edge of the left hole is rigidly fixed and loads are applied at the edge of the right hole (point A). There are two load cases; one includes a horizontal tension force and the other a horizontal and a vertical force.[2] The thickness $t = 0.001$ m and $E = 200$ GPa are assumed. The best optimum is the design objective.

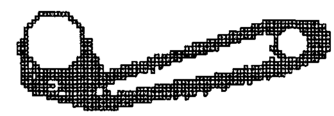
In case 1, the full design area is divided by a mesh of 76×38 and the initial design is shown in Fig. 12. The maximum design, reached at the 32nd iteration, has a weight of 26%. Figure 13 gives the optimum topologies obtained by ESO and BESO. In case 2, a finer finite element mesh of 100×50 is used. The weight of the maximum design is 36% in BESO. Final optimum designs are given in Fig. 14. It is seen in Figs. 13 and 14 that the corresponding topologies obtained by two methods are similar. The savings in solution time because of BESO is very significant, as shown in Table 3. This is because the maximum and optimum designs in the two cases only cover a very small part of the full design area, and thus the iteration is far less in BESO.

Table 4 Comparison of ESO and BESO (angle piece)

Type of optimization	Iterations	W/W_0, %	C, N · m	W_{obj}	Time, h
ESO	70	28.29	44.65	95.51	4.5
BESO	97	28.29	43.47	91.21	3



ESO: $W_{opt}/W_0 = 16.5\%$, $W_{obj} = 1.400$



BESO: $W_{opt}/W_0 = 14.5\%$, $W_{obj} = 1.413$

Fig. 14 Optimum topologies (lever arm under loading in tension and bending).

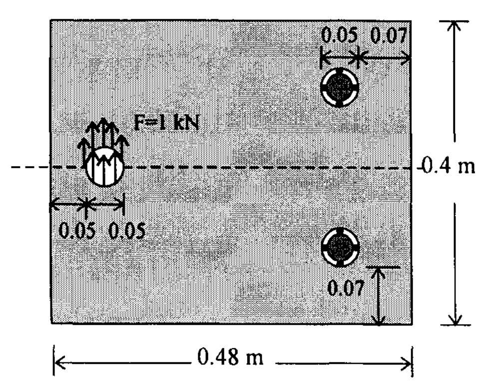


Fig. 15 Design area of an angle piece.

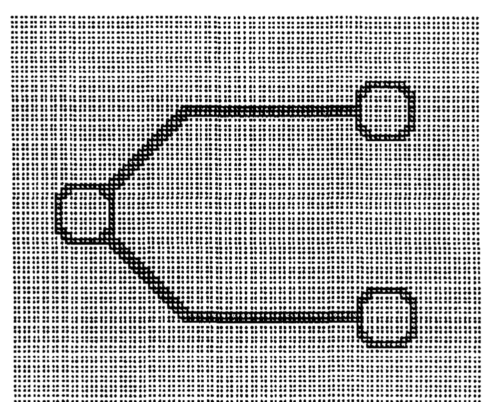
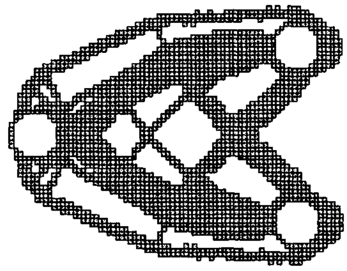


Fig. 16 Initial design (angle piece).

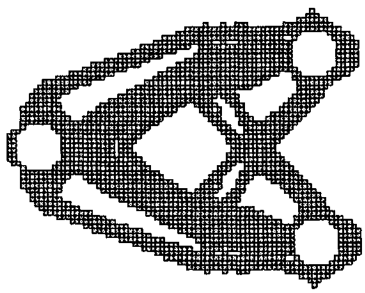
Example 4: Angle Piece

The design area, loading and boundary conditions are given in Fig. 15. The thickness $t = 0.001$ m and $E = 200$ GPa are assumed. The best optimum of an angle piece is to be sought within the rectangular area. The area is divided by a mesh of 96×80. Figure 16 gives the initial design. The maximum design in BESO has a weight of 42%. The topologies obtained by ESO and BESO are shown in Fig. 17 and the results are summarized in Table 4. BESO results in a smaller objective weight and at the same time is more efficient in this example.

There are several points worth noting regarding the preceding stiffness and displacement optimization. First, the sensitivity number α and displacement extrapolation play essential roles in BESO. As the sensitivity number is derived on the basis of optimality criteria procedure,[11] BESO is generally valid regardless of the structural system. To extend the proposed method to other problems such as



ESO: $W_{opt}/W_0 = 28.29\%$, $W_{obj} = 95.51$



BESO: $W_{opt}/W_0 = 28.29\%$, $W_{obj} = 91.21$

Fig. 17 Optimum topologies (angle piece)

the plate bending and three-dimensional continua needs modification on the displacement extrapolation. Although a simple graphical demonstration of an extrapolation like Fig. 2 may not be available in these problems, a general method using the shape function of the finite element can be employed to extrapolate the unknown displacement.

Second, the BESO procedure involves the concept of objective weight. For problems where the stiffness matrix is proportional to the plate thickness or its zth order, or generally speaking, the constraint is the linear function of the reciprocal design variable or its zth order, the objective weight can be easily defined and calculated. There are some cases where this relationship does not hold true, such as the three-dimensional continua. As the objective weight is mainly for determining the maximum design and is not indispensable for BESO, an easiest solution to the problem is that the designer specifies a maximum design before hand. A relatively larger value (60%) can serve the purpose.

Third, the extension of the BESO method to other constraints is straightforward with only minor modifications on the calculation of sensitivity number. For example, the computer codes for stiffness and frequency optimization can be similar and the major difference is the expression for the sensitivity number.[9] This makes the BESO method rather flexible and adaptable in the computer implementation.

Conclusions

The BESO method is tested on some stiffness and displacement optimization problems. Its feasibility is established by the satisfactory agreement of results of BESO and those of ESO, which also demonstrates the validity of the evolutionary algorithm. Differences in optimal topologies obtained by two methods are observed, which reflects the feature of the evolutionary method as a ground structure approach. BESO can yield better numerical solutions as it can suppress the problem of prematurely removed elements that may be encountered in the ESO. BESO is computationally more efficient than ESO in most cases, particularly when the maximum design and the optimum topology only cover a low proportion of the full design area.

Although the study in this paper focuses on two-dimensional problems under the plane stress condition, the proposed method is not restricted to this kind of structure. The procedure of BESO is general and can be applied to problems of different structural systems as well as different optimization constraints.

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